CS 6770 Natural Language Processing

Statistical Language Modeling

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Overview

1. Introduction
2. n-gram Language Models
3. Generation with Bi-gram Models
4. Smoothing Techniques
5. Language Model Evaluation

Introduction

Word Prediction with Context

Consider the example, what words are likely to follow

```
Please turn your homework ...
```

[Jurafsky and Martin, 2019]

Word Prediction with Context

Consider the example, what words are likely to follow

```
Please turn your homework ...
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Although we do not know the actual word in the original text, we have a good sense about what of these following words are likely to follow

- `in`
- `over`
- `refrigerator`
- `the`

[Jurafsky and Martin, 2019]

Mathematical Formulation

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$$P(X_t \mid X_1, \ldots, X_{t-1}) \quad \text{or} \quad P(X_t \mid \boldsymbol{X}_{1:t-1}) \tag{1}$$

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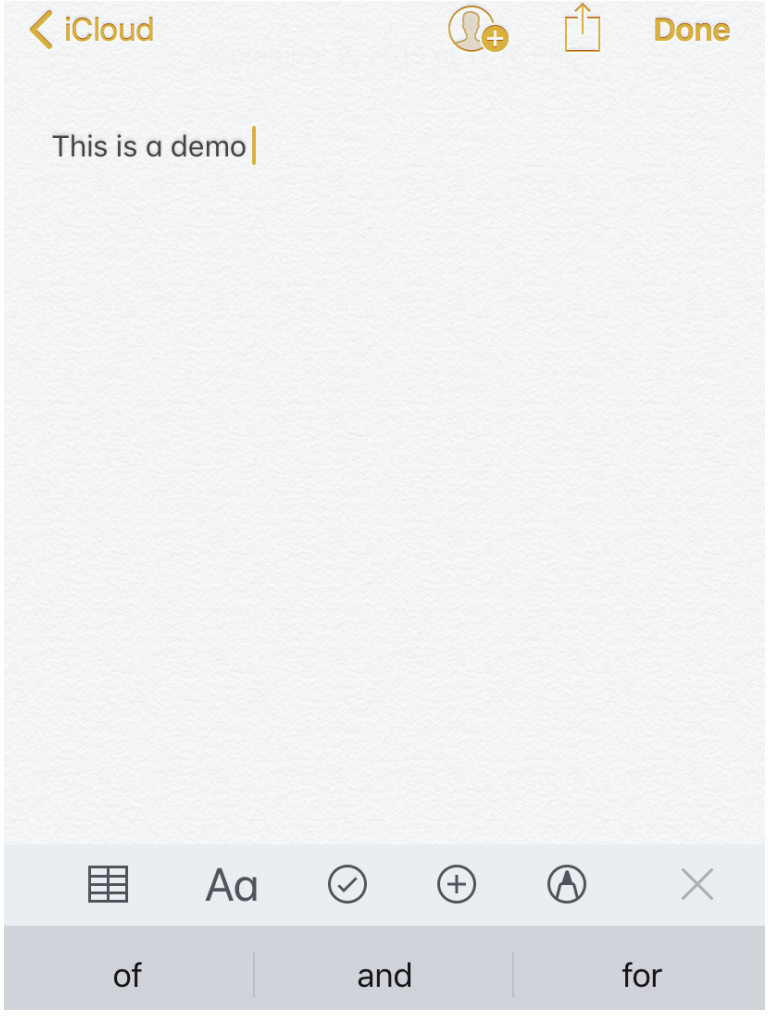
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- The topics in this and the next lectures will offer two modeling methods in equation 1.

Word Prediction in Input Methods

Input methods use language models to predict the next likely words, to speed up the typing



Writing a Poem?

Trevor Noah and Amanda Gorman writing poems with the input methods on their phones

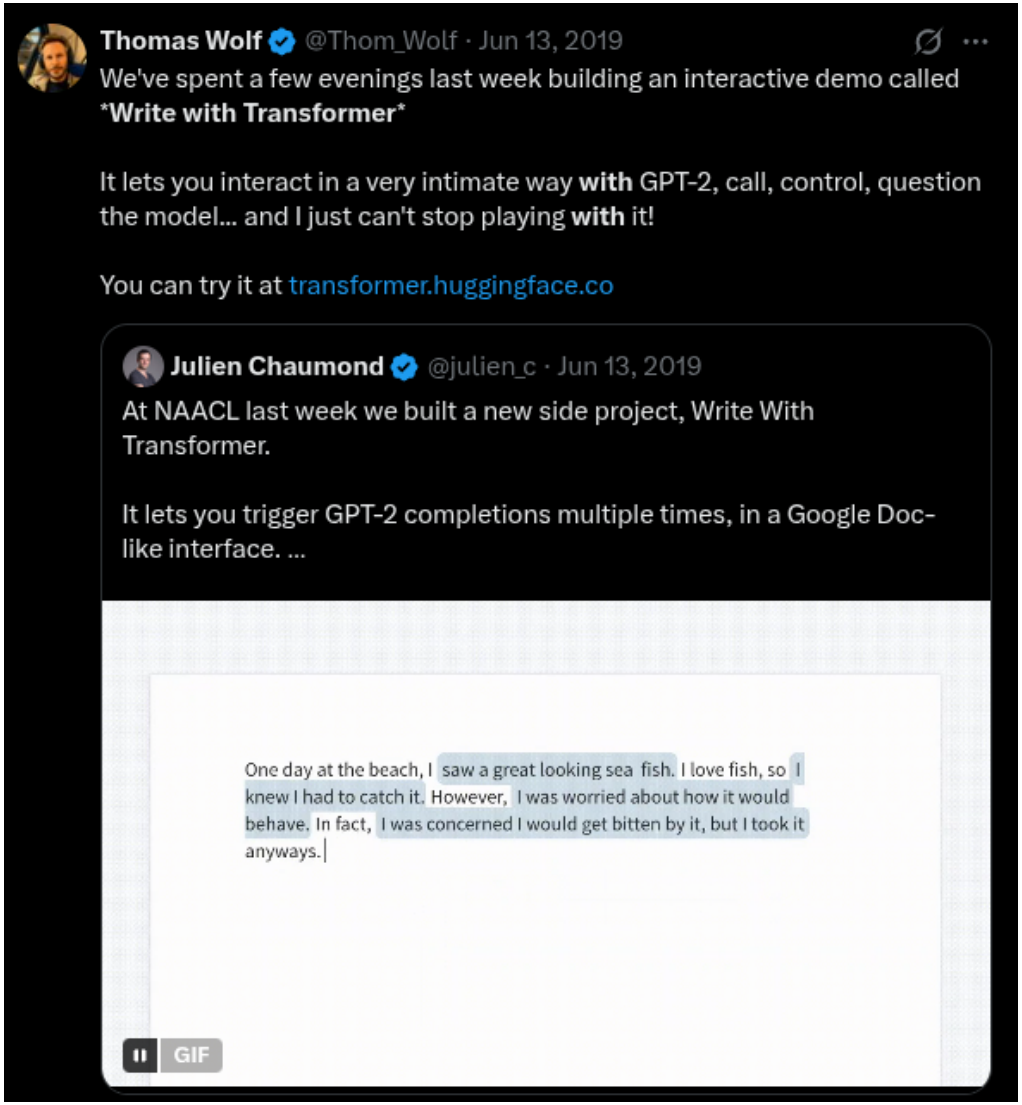


Figure: The Daily Social Distancing Show: Bonus Track feat. Amanda Gorman

Link

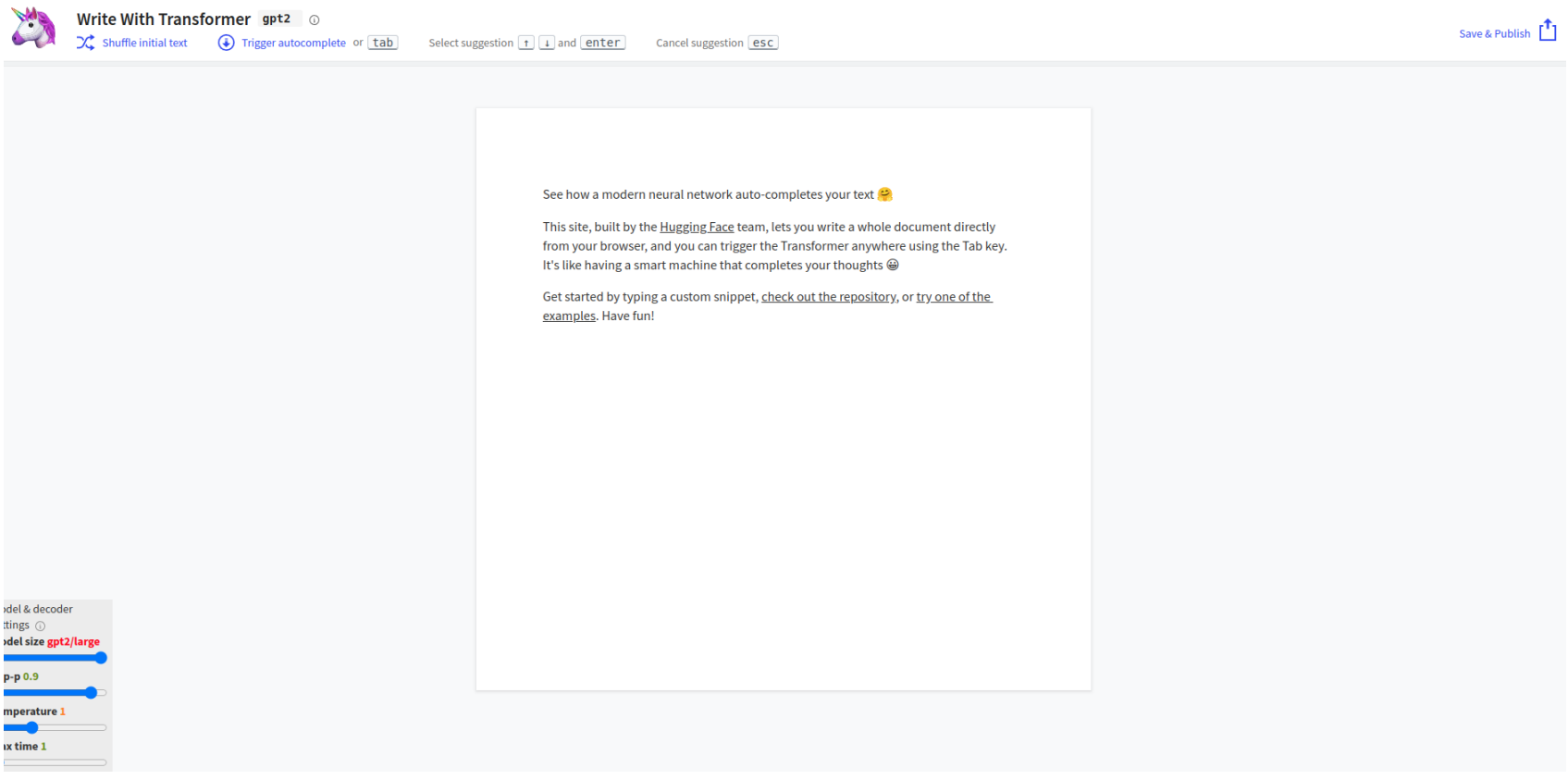
Write with Transformer

The original Twitter post about this demo:



Write with Transformer

A demo supported by GPT-2



Link

Joint Probability and Chain Rule

Given the conditional probability $P(X_t \mid \boldsymbol{X}_{1:t-1})$, to evaluate the qualty of a text, we need the chain rule in probability to factorize the joint probability $P(\boldsymbol{X}_{1:t})$ into a series of conditional probabilities.

$$P(X_1, X_2, \cdots, X_t) \quad = \quad P(X_1)P(X_2, \cdots, X_k \mid X_1) \tag{2}$$

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&= P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1, X_2)\cdots \\
&\quad P(X_t \mid X_1, \cdots, X_{t-1}) \\
&= \prod_{i=1}^{k} P(X_i \mid X_1, \ldots, X_{i-1}) \qquad (2)
\end{aligned}
$$

Speech Recognition



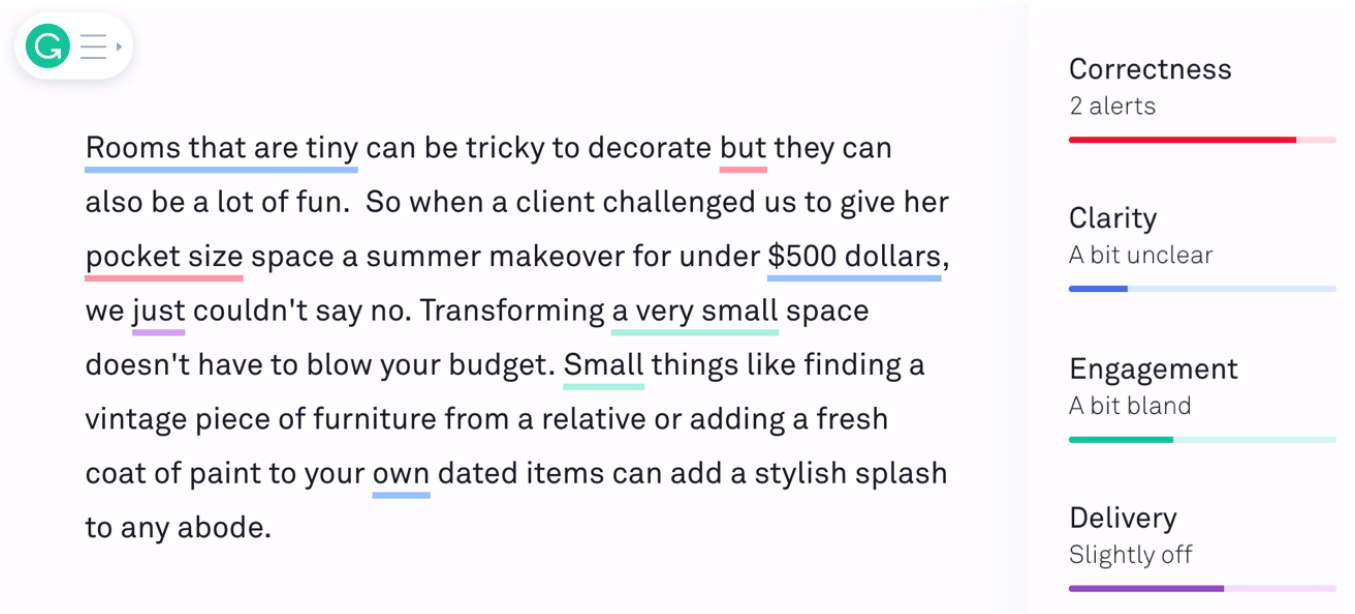
Given a voice signal, a language model in speech recognition will evaluate the likelihood of decoded texts

$$P(\text{I saw a van}) \gg P(\text{eyes awe of an}) \tag{3}$$

[Jurafsky and Martin, 2019]

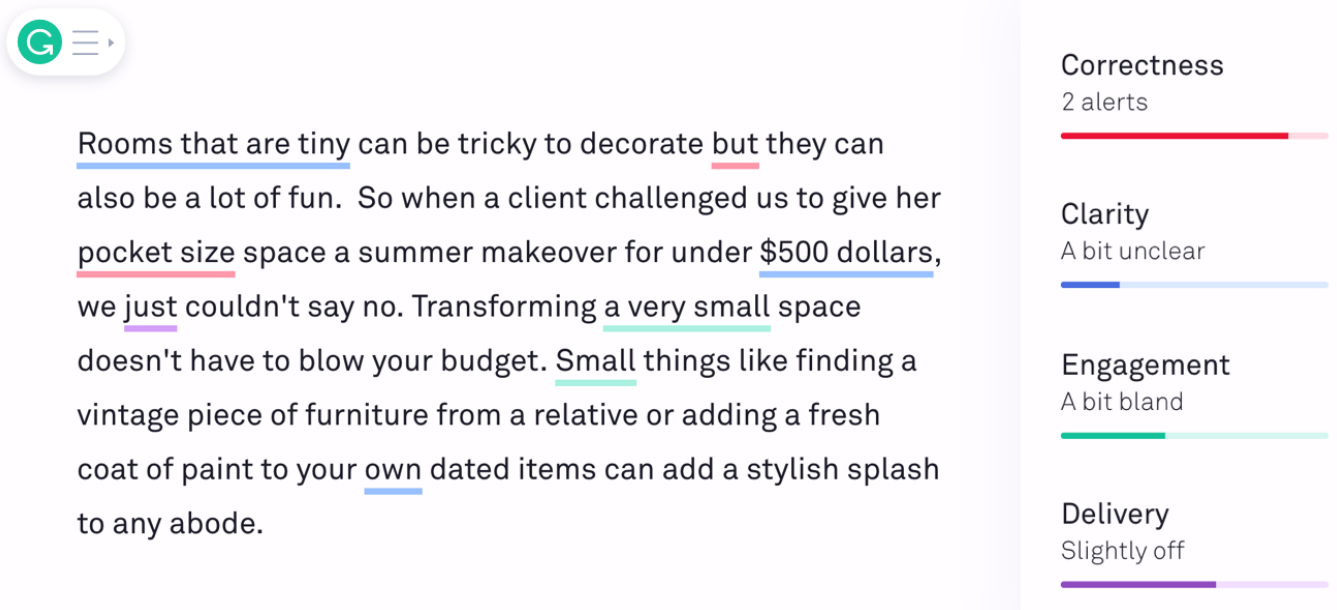
Writing Assistant

Grammarly:



Writing Assistant

Grammarly:



A good writing assistant system involves two tasks

- evaluate the quality of a text
- generate revision suggestions

A language model cannot provide support to all functions directly, but is a critical component in the backend system

Applications

- Generative tasks: predicting the next word given a context
  - Word prediction
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  - ...

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- Generative tasks: predicting the next word given a context
  - Word prediction
  - Text generation
  - ...
- Discriminative tasks: evaluating the quality of texts
  - Speech recognition
  - Machine translation
  - Document summarization
  - ...

n-gram Language Models

Problem Definition

Given a vocab $\mathcal{V}$ that contains all the possible word types, then the prediction of X_t can be formulated as

$$P(X_t \mid X_1, \ldots, X_{t-1}) = _____ \tag{4}$$

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- it consider the entire context from the very first word X_1 to the previous word X_{t-1}
  - The main topic of this section

Parameter Estimation

With a collection of texts as training examples, the simple method of estimating the probabilities is using maximum likelihood estimation.

- In statistics, the parameter estimation (MLE) of a Bernoulli distribution is intuitive

$$\hat{P}(X = 1) = \frac{\sum_{i=1}^{N} \delta(x_i, 1)}{N} = \frac{c(X = 1)}{N} \tag{5}$$

  where $c(X = 1)$ is the number of observations with value 1

[Collins, 2017]

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- Similarly, to estimate the conditional probability $P(X_t \mid \boldsymbol{X}_{1:t-1})$, we have

$$\hat{P}(X_t = x_t \mid \boldsymbol{X}_{1:t-1} = \boldsymbol{x}_{1:t-1}) = \frac{c(\boldsymbol{x}_{1:t})}{c(\boldsymbol{x}_{1:t-1})} \tag{6}$$

  where $c(\boldsymbol{x}_{1:t-1})$ is the number that text $\boldsymbol{x}_{1:t-1}$ appears in the training examples, and $c(\boldsymbol{x}_{1:t})$ is the number that text $\boldsymbol{x}_{1:t}$ appears in the training examples.

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Parameter Estimation (Cont.)

Imagine we have a huge collection of texts for parameter estimation

- With the sentence "the dog barks"

$$\hat{P}(X_3 = \texttt{barks} \mid \boldsymbol{X}_{1:2} = \texttt{the dog}) = \frac{c(\boldsymbol{X}_{1:3} = \texttt{the dog barks})}{c(\boldsymbol{X}_{1:2} = \texttt{the dog})} \quad (7)$$

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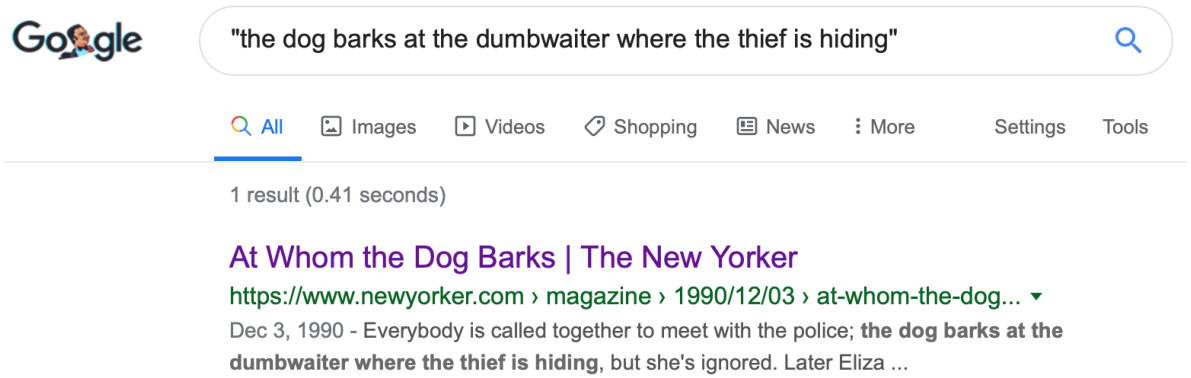
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- With the sentence "`the dog barks at the dumbwaiter where the thief is hiding`"

$$\begin{aligned}\hat{P}(X_{11} = \texttt{hiding} \mid \boldsymbol{X}_{1:10} = \texttt{the dog} \cdots \texttt{is}) \\ = \frac{c(X_{1:11} = \texttt{the dog} \cdots \texttt{is hiding})}{c(X_{1:10} = \texttt{the dog} \cdots \texttt{is})}\end{aligned} \tag{8}$$

Parameter Estimation (Cont.)

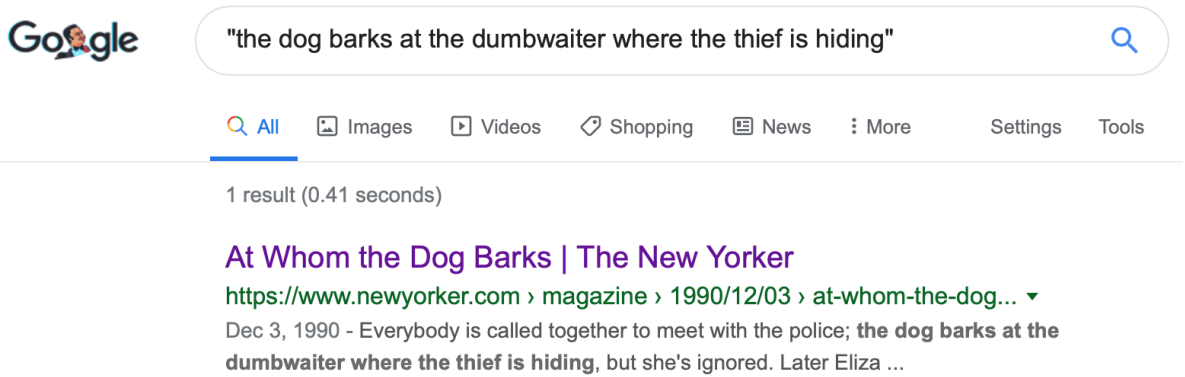
For this specific sentence, we only one training example even if we collect all the texts from the Internet[1]



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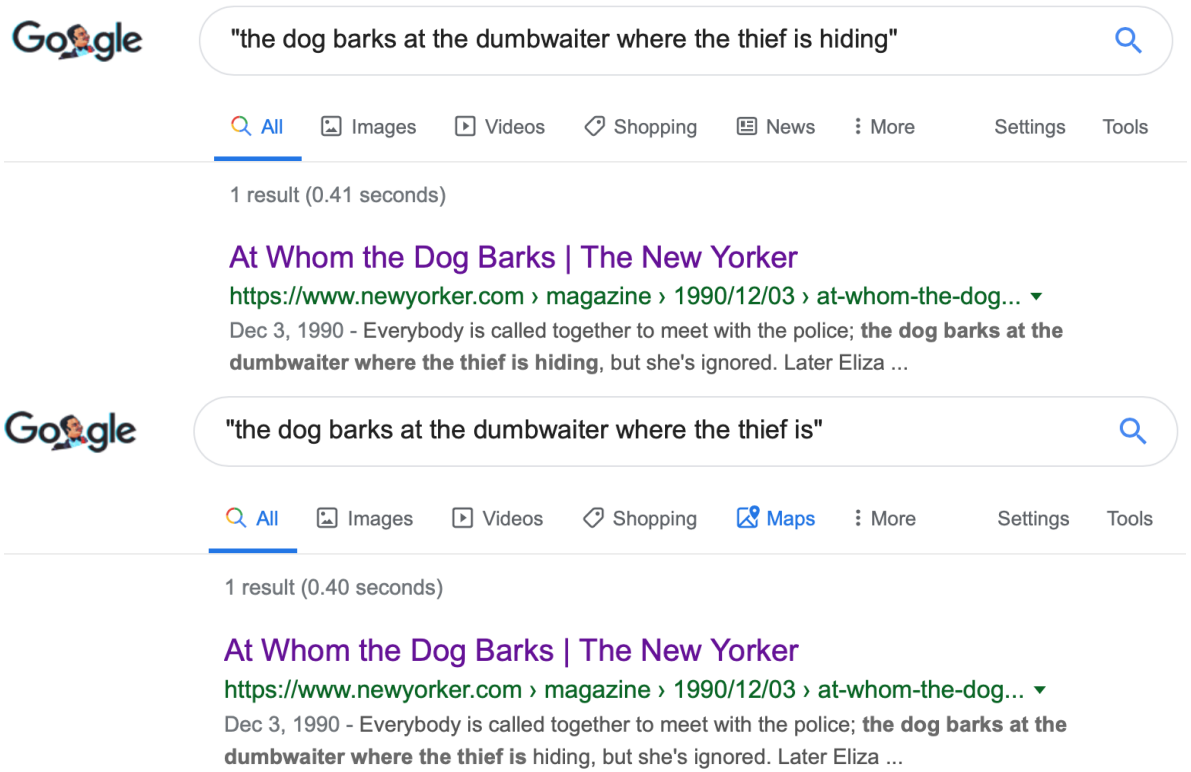


A dumbwaiter is a small elevator designed for transporting goods, not people, between floors.

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- Comments: the tradeoff between prediction power and number of parameters
  - It has extremely limited prediction power
  - Number of parameters: $V = \mathcal{O}(|\mathcal{V}|)$

Bi-gram Models

- To find a good balance between the prediction power and parameter estimation challenge, we can limit the contextual information used in a language modeling.
- Bi-gram model: uses only one word X_{t-1} from the previous context to predict the current word X_t

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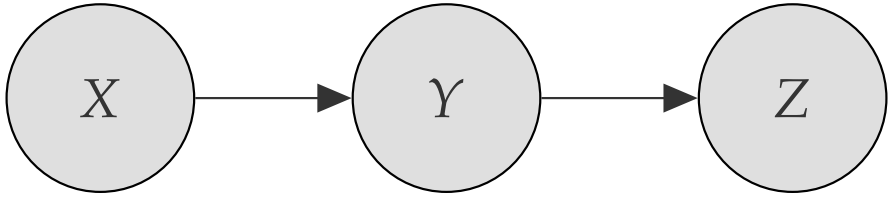
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- In probabilistic modeling, a bi-gram model is an application of the first-order Markov model

Markov Property

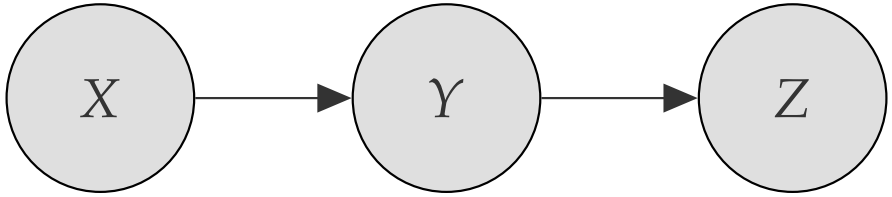
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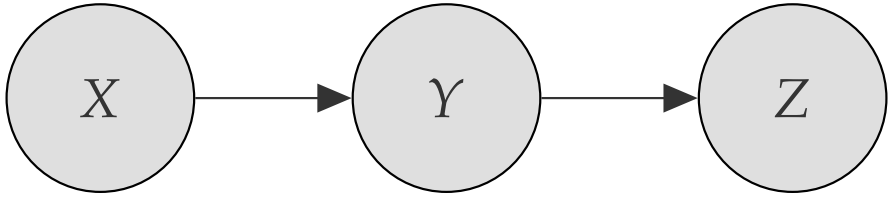
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and also the joint probability

$$\begin{aligned} P(X_1, \ldots, X_t) \quad \approx \quad & P(X_t \mid X_{t-1}) \cdot P(X_{t-1} \mid X_{t-2}) \cdots \\ & P(X_2 \mid X_1) \cdot P(X_1) \end{aligned} \tag{15}$$

Special Tokens

Consider the application of using a bi-gram model

$$P(\text{the dog barks}) = P(\text{the}) \cdot P(\text{dog} \mid \text{the}) \; P(\text{barks} \mid \text{dog})$$

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Factorization with special tokens:

$$P(\square \text{ the dog barks } \blacksquare) = P(\text{the} \mid \square) \cdot P(\text{dog} \mid \text{the}) \; P(\text{barks} \mid \text{dog}) \cdot P(\blacksquare \mid \text{barks})$$

Example: Parameter Estimation

Example sentences

- □ `I am Sam` ■
- □ `Sam I am` ■
- □ `I do not like green eggs and ham` ■

[Jurafsky and Martin, 2019]

Example: Parameter Estimation

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Some of the probabilities:

$$\hat{P}(\text{I} \mid \square) = \frac{2}{3} \quad \hat{P}(\blacksquare \mid \text{Sam}) = \frac{1}{2} \quad \hat{P}(\text{do} \mid \text{I}) = \frac{1}{3} \tag{16}$$

[Jurafsky and Martin, 2019]

Issues with a Fixed Vocabulary

- $P(X_t \mid X_{t-1})$ is defined a fixed vocabulary, for normalization purpose

$$P(X_t \mid X_{t-1}) = \frac{c(X_{t-1}, X_t)}{\sum_{X' \in \mathcal{V}} c(X_{t-1}, X')} \tag{17}$$

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- Issues with a fixed vocabulary
  - Unknown words: word x is not in the vocabulary
  - Zero probability: word combination (x, x') never appears in the training set

Unknown Words

Replace all words that are not in the vocab with a special token UNK.

For example

- Original text: "`the dog barks at the dumbwaiter where the thief is hiding`"
- After preprocessing: "`the dog barks at the` UNK `where the thief is hiding`"

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Question

Can we simply ignore the unknown words? For example, what if the preprocessed text is

"`the dog barks at the  where the thief is hiding`"

Tri-gram Models

We can extend the conditional probability to depend on previous two tokens

$$P(X_t \mid X_1, \ldots, X_{t-1}) \approx P(X_t \mid X_{t-2}, X_{t-1}) \tag{18}$$

Comments

- More dependency leads to more accurate predictions
- Parameter estimation

$$\hat{P}(X_t \mid X_{t-2}, X_{t-1}) = \frac{c(\mathbf{X}_{t-2:t})}{c(\mathbf{X}_{t-2:t-1})} \tag{19}$$

Number of Parameters

- Uni-gram model
  - Ignore context words completely $P(X_t \mid \boldsymbol{X}_{1:t-1}) \approx P(X_t)$
  - Number of parameters $\mathcal{O}(|\mathcal{V}|)$

	X_1	$\cdots$	$X_{\lvert\mathcal{V}\rvert}$
$P(X_t)$			

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- Bi-gram model
  - Use only the adjacent word $P(X_t \mid \boldsymbol{X}_{1:t-1}) \approx P(X_t \mid X_{t-1})$
  - Number of parameters $\mathcal{O}(|\mathcal{V}|^2)$

$P(X_t \mid X_{t-1})$	X_1	$\cdots$	$X_{\lvert\mathcal{V}\rvert}$
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X_1			
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- Tri-gram model
  - Use two preceding words $P(X_t \mid \boldsymbol{X}_{1:t-1}) \approx P(X_t \mid X_{t-2}, X_{t-1})$
  - Number of parameters $\mathcal{O}(|\mathcal{V}|^3)$

Generation with Bi-gram Models

Demo Code

- A bi-gram model with no smoothing
- Training with the dataset from the arXiv paper abstracts
- Generating by *randomly* sampling from this bi-gram model

Demo Code

- A bi-gram model with no smoothing
- Training with the dataset from the arXiv paper abstracts
- Generating by *randomly* sampling from this bi-gram model

Checkout the demo code for some examples

Smoothing Techniques

High-order Markov Models

A motivating example:

```
The printer on the 5th floor of Rice hall crashed
```

High-order Markov Models

A motivating example:

`The printer on the 5th floor of Rice hall crashed`

n-gram Language Models

- Uni-gram: $P(X_t)$
- Bi-gram: $P(X_t \mid X_{t-1})$
- Tri-gram: $P(X_t \mid X_{t-2}, X_{t-1})$
- 4-gram: $P(X_t \mid X_{t-3}, X_{t-2}, X_{t-1})$
- 5-gram: $P(X_t \mid X_{t-4}, X_{t-3}, X_{t-2}, X_{t-1})$

Discounting

It is the same method used in parameter estimation of naive Bayes classifiers

$$P(X_t \mid X_{t-1}) = \frac{c(X_{t-1}, X_t) + \alpha}{c(X_{t-1}) + \alpha V} \tag{20}$$

where $\alpha > 0$ is a hyper-parameter.

Linear Interpolation

Estimate the following three models with MLE:

- Uni-gram: $P(X_t)$
- Bi-gram: $P(X_t \mid X_{t-1})$
- Tri-gram: $P(X_t \mid X_{t-2}, X_{t-1})$

Linear Interpolation

Estimate the following three models with MLE:

- Uni-gram: $P(X_t)$
- Bi-gram: $P(X_t \mid X_{t-1})$
- Tri-gram: $P(X_t \mid X_{t-2}, X_{t-1})$

Then, the new probability of X_t given X_{t-2} and X_{t-1} is

$$\begin{aligned} P_{LI}(X_t \mid X_{t-2}, X_{t-1}) &= \lambda_1 \cdot P(X_t) + \lambda_2 \cdot P(X_t \mid X_{t-1}) \\ &\quad + \lambda_3 \cdot P(X_t \mid X_{t-2}, X_{t-1}) \end{aligned} \tag{21}$$

$\{\lambda_i\}$ are learned with a held-out corpus (a development set).

Language Model Evaluation

Sentence Evaluation (I)

Evaluation with joint probabilities

$$P(\text{I love black coffee}) \quad \text{vs.} \quad P(\text{black coffee pleases me}) \tag{22}$$

Direct comparison between the probabilities will tell us which sentence is more *fluent*.

Sentence Evaluation (II)

Limitation of comparing joint probabilities directly

$$P(\text{I love black coffee}) \quad \text{vs.} \quad P(\text{I like black coffee very much}) \tag{23}$$

Due to the *length difference*, the second probability may always be smaller than the first.

Likelihood

- Test data: including the special tokens

$$\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_M$$

- Likelihood

$$\text{Log-lik}(\{\boldsymbol{x}_{m=1}^{M}\}) = \log_2 \prod_{m=1}^{M} \prod_{t=1} P(x_{m,t} \mid \boldsymbol{x}_{m,1:t-1}) \quad (24)$$

$$= \sum_{m=1}^{M} \sum_{t=1} \log_2 P(x_{m,t} \mid \boldsymbol{x}_{m,1:t-1}) \quad (25)$$

- Factors
  - Number of the tokens
  - No intuitive explanation

Perplexity

The definition of perplexity is

$$\text{Perplexity} = 2^{-\frac{1}{T}\text{Log-lik}(\{\boldsymbol{x}_{m=1}^{M}\})} \tag{26}$$

where T is the total number of the log probabilities in Log-lik$(\{\boldsymbol{x}_{m=1}^{M}\})$.

Special Case

- An impossible case

$$P(x_t \mid \boldsymbol{x}_{1:t-1}) = 1 \tag{27}$$

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- Perplexity

$$\begin{aligned} \text{Perplexity} &= 2^{-\frac{1}{T}\sum_{k=1}^{M}\sum_{m=1}\log_2 1} \\ &= 2^0 \\ &= 1 \end{aligned} \tag{28}$$

Special Case (II)

- A trivial case

$$P(x_t \mid \boldsymbol{x}_{1:t-1}) = \frac{1}{|\mathcal{V}|} \tag{29}$$

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- A trivial case

$$P(x_t \mid \boldsymbol{x}_{1:t-1}) = \frac{1}{|\mathcal{V}|} \tag{29}$$

- Perplexity

$$\begin{aligned} \text{Perplexity} &= 2^{-\frac{1}{T}\sum_{k=1}^{M}\sum_{m=1}\log_2 \frac{1}{|\mathcal{V}|}} \\ &= 2^{-\frac{1}{T}(T\cdot\log_2 \frac{1}{|\mathcal{V}|})} \\ &= 2^{-\log_2 \frac{1}{|\mathcal{V}|}} \\ &= |\mathcal{V}| \end{aligned} \tag{30}$$

Typical Values of Perplexity

- $|\mathcal{V}| = 50K$
- A uni-gram model: Perplexity = 955
- A bi-gram model: Perplexity = 137
- A tri-gram model: Perplexity = 74

Lower is better

[Collins, 2017]

A Few Comments on Perplexity

Perplexity

- is an intrinsic evaluation measurement

A Few Comments on Perplexity

Perplexity

- is an intrinsic evaluation measurement
- is not necessarily correlated with the performance of
  - e.g., lower perplexity does not mean better translation (wrt BLEU score)
- is not directly comparable even on the same test data
  - you need the exactly same input for comparison

Reference



Collins, M. (2017).
Natural language processing: Lecture notes.



Jurafsky, D. and Martin, J. (2019).
Speech and language processing.